Part II Tripos Bookwork

FCM

1. Write down Green's and Stokes' theorem.
2. Write down the *Sokhotski–Plemelj relations*. Derive a function from its real part.
3. Write down *Euler's product formula* for the Γ-function; hence derive its *Weierstrass canonical product formula*.
4. List all functional equations for the Γ-, B- and ζ-functions. Write down their integral representations (noting any analytic continuations) with their domains and singularities.
5. Define *elliptic functions* and list their properties. Familiarise with the *Weierstrass $\wp$-function*.
6. Write down the condition on $p(z)$ and $q(z)$ for ∞ to be an ordinary point.
7. Write down the general result for transformations of the Papperitz symbol. Write down the *canonical Papperitz symbol* with the hypergeometric equation. Write down the hypergeometric series and derive its integral representation using the B-function.

CD

1. State the *principle of least action*.
2. *Define *holonomic constraints*. Define the *continuous symmetry* of the Lagrangian, and write down a conserved quantity (by Noether's theorem).
3. Write down the matrix equation in a perturbation analysis. Define the equilibrium point.
4. Derive an expression for the *moment of inertia tensor* from the kinetic energy definition. Derive the *parallel axes theorem*.
5. List relations between space and body frame bases. Derive *Euler's equations* for a rotating rigid body.
6. Define *Euler's angles* by drawing a picture. Express the instantaneous angular velocity in the body frame.
7. Write down the *symplectic condition* for a canonical transformation.
8. Explain the term *action-angle variables*. Write down the integral expression for the action variable.
9. Explain the term *adiabatic invariant*. Give an example for an integral system.

C

1. Using the *cosmological principle*, derive *Hubble's law*.
2. Write down the particle species, relativistic and/or interacting, at $\lesssim 5 \times 10^9\,\mathrm{K}$, $5 \times 10^9 \sim 10^{10}\,\mathrm{K}$ and $\gtrsim 10^{10}\,\mathrm{K}$.
3. Write down the scale factor-time, density-time, and temperature-time relations in matter- and radiation-dominated eras.
4. State the results from BBN on the abundance of $^4\mathrm{He}$.
5. Review the particle survival differential equations.
6. State the *flatness problem* and the *horizon problem*. Explain the term *inflation*, its conditions and how it solves these problems.
7. Write down the *slow-roll conditions*. Define the *e-fold* number.
8. (!)*Familiarise with the linearisation leading to the Jeans stability equation(s).

AM

1. Write down *Watson's lemma*.
2. Define the terms *asymptotic relation*, *asymptotic sequence*, *asymptotic expansion* and *optimal truncation*.
3. Describe how the *connection formula* in WKB(J) approximation is obtained:

$$\int_a^b \sqrt{2m\,|E - V(x)|}dx = (n + \frac{1}{2})\pi\hbar.$$

   Derive the formula for the eigen-energies on the interval $[0,\, L]$ with the boundary conditions $y(0) = y(L) = 0$.

IS

1. Define a flow map and its generator, and write down its properties (also for a one-parameter group of transformations). Write down the commutator of two vector fields, and the Poisson bracket in two ways. Prove $[\mathbf{v}_f,\, \mathbf{v}_g] = -\mathbf{v}_{\{f,g\}}$.
2. Describe the process of solving inverse scattering with a reflectionless potential with the help of the GLM-equation. You may use $\det(I + \epsilon M) = 1 + \epsilon \mathrm{tr} M + o(\epsilon)$ and thus prove $(\det A)' = \det A\, \mathrm{tr}(A^{-1}A')$.
3. Prove the eigenvalues of the operator L in a *lax pair* $(L,\, A)$ is time-independent. Deduce that if ψ is an eigenvector, then so is $\psi' := \psi_t + A\psi$ with the same eigenvalue. Recall zero-curvature equations are compatibility conditions on a lax pair.

4. Define the *Hamiltonian form* of an evolution equation. Define the *functional derivative*, and write down its formula. Recall bi-Hamiltonian system, such as the KdV hierarchy, have infinitely many Hamiltonians in involution.

5. Prove $\frac{dI}{dt} = \{I,\, H\}$.

6. Define the *Lie point symmetry* of a PDE, and deduce an equivalent condition. Write down the generator of a one-parameter group of transformations. Write down the *prolongation formula* (or derive it).

PQM

1. Derive the *completeness relation.* Write down the *canonical inner products* $\langle x|p\rangle$, $\langle p|x\rangle$, $\langle x|x'\rangle$, $\langle p|p'\rangle$. Hence derive $\langle x|\hat{p}|x'\rangle$ and $\langle p|\hat{x}|p'\rangle$, and thus $\langle x|\hat{p}|\psi\rangle$ and $\langle p|\hat{x}|\psi\rangle$, $\langle p|V(\hat{x})|\psi\rangle$. Derive the Schrödinger equation in momentum space.

2. Recall $a^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$, $a\,|n\rangle = \sqrt{n}\,|n-1\rangle$ and $|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}\,|0\rangle$.

3. Write down the perturbation formulae to first and second orders in the non-degenerate case. Write down the perturbation formula in the degenerate case. Derive both results.

4. Write down properties of *Pauli matrices.* Write down some general commutation relations.

5. Recall $J_\pm\,|j,\, m\rangle = \hbar\sqrt{(j\mp m)(j\pm m+1)}\,|j,\, m\pm 1\rangle$. Write down an inequality for j and m, and in the case of addition of angular momentum, an inequality for J, L and S.

6. Practise some *Clebsch-Gordon coefficient* calculations. What is the total number of states for $j_1,\, j_2$?

7. Define the *symmetry* of a quantum system. Recall that $\hat{x}$ acts like $i\hbar\partial_{\hat{p}}$ inside a commutator, so could deduce $U(a) = e^{-i\hat{p}a/\hbar}$ for translation.

8. Define the *time-evolution operator*, the *interaction picture* and thus derive its equation of motion. Hence derive the first order transition rate.

9. *Familiarise with the concept of a *density operator.* Distinguish between *pure* and *entangled states.*

FD II

1. Establish the constitutive relations on the assumption of isotropy, incompressibility and symmetry.

2. Write down the *mechanical energy* equation and explain each term. Write viscous dissipation per unit volume as $2\mu e_{ij}e_{ij}$.

3. State the simple properties of Stokes flows.

4. Prove the following lemma. State and derive the *minimal dissipation theorem* and the *reciprocal theorem.*

Lemma. $\int_V 2\mu e^s_{ij} e_{ij} dV = \int_{\partial V} \sigma^s_{ij} n_j u_i dS.$

5. Write down the assumptions and the simplified equations for unidirectional flows.
6. Write down the assumptions and the simplified equations for thin layer approximations.
7. Write down the assumptions and the simplified equations for boundary layer approximations.
8. Derive the vorticity equation.
9. Linearise the kinematic, dynamic boundary conditions for Kelvin-Helmholtz instability. *Discuss the difference between Taylor-Rayleigh and Kelvin-Helmholtz instabilities.
10. (!)Familiarise with the following examples: flow past a sphere, flow in a corner, gravitational spreading of droplets, wall with suction, Burgers' vortex, 2-d momentum jets and the rise of a spherical bubble.

SP

1. Describe three statistical mechanical ensembles. For each ensemble, write down the partition function and relevant thermodynamic quantities. Explain why all ensembles agree in the *thermodynamic limit.* Explain the difference between the (semi-)classical and the quantum description in temperature limits.
2. Write down the *Boltzmann distribution*, the *Maxwell distribution*, the *Planck distribution*, the *Bose-Einstein distribution* and the *Fermi-Dirac distribution.*
3. Write down the *van der Waal's equation of states.* Give the physical significance of the parameters a, b in the vdW equation of states.
4. Recall that fugacity $\zeta \equiv e^{\beta\mu} \sim T^{-3/2}$. Determine the conditions on ζ in the high and low temperature limits. Define the *critical temperature* for Bose-Einstein condensation.
5. Define the *polylogarithm function* and state its properties. Write down the *Sommerfeld expansion.*
6. Write down the definitions and differentials of five thermodynamic potentials. Think of relevant thermodynamic quantities in terms of these potentials.
7. *Define *adiabatic*, *quasi-static* and *reversible* processes. Describe a *Carnot cycle.*
8. Describe the *Maxwell construction.*
9. Write down the average spin per particle in terms of the canonical partition function. Derive a *mean field theory* in the Ising model.

GR

1. Define the *Christoffel symbol*. Write down the covariant derivatives of second rank tensors of valence (2,0), (0,2) and (1,1).

2. Write down the coordinate transformations for tensors and the Levi-Civita connection.

3. Establish the following results in *geodesic deviation*: $\nabla_V T^a = \nabla_T V^a$; $\nabla_V \nabla_T Q^a - \nabla_T \nabla_V Q^a = R^a{}_{bcd} Q^b V^c T^d$; $\frac{D^2}{D\tau^2} V^a = R^a{}_{bcd} T^b T^c V^d$.

4. Write down the *Ricci identity* and the *Riemann tensor* with all its symmetries. Write down the *Einstein field equations* with a cosmological constant.

5. *Familiarise with the linearisation of the Riemann tensors in the perturbation of the Minkowski metric, as well as the *de Donder* gauge.

6. (!)Familiarise with the following examples: perihelion precession, light deflection.

W

1. Write down the linearised perturbation equations for quantities $\tilde{p}$, $\tilde{\rho}$, u, ϕ, $\mathbf{I}$. Write down thermodynamic relations. Define the terms in the wave energy equation, obtained from $\mathbf{u} \cdot \text{c.o.mom.} + \frac{\tilde{p}}{\rho}\text{c.o.m.}$.

2. Write down the formula for calculating the time average of a quadratic quantity.

3. State the one-dimensional gas equations. Derive the Riemann form of equations and the Riemann invariants. Repeat for shallow-water equations (given; $\gamma = 2$).

4. Write down the *Rankine-Hugoniot relations* for 1-dimensional gas and shallow water waves.

5. Write down the *constitutive relation* for elastic solids. State the interfacial boundary conditions for elastic waves.

6. Write down the compressional and shear wave speeds for elastic solids in terms of the Lamé moduli. Distinguish between P, SV, SH waves.

7. Derive the linearised equations for internally stratified water waves.

8. Derive the *ray-tracing equations*. Recall *Fermat's principle* for slowly varying, steady, non-dispersive media.

9. Relate dispersion relations in a medium with and without mean flow.

10. (!)Familiarise with the following examples: Mach cones, duck waves, waves approaching a beach, Love waves, Rayleigh waves, geometric waveguides.